When Witnesses Defend: A Witness Graph Topological Layer for Adversarial Graph Learning.

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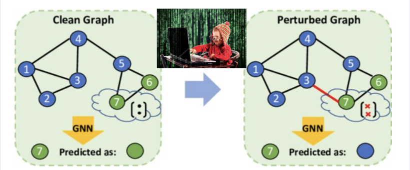
Adversarial attack: a Modern Challenge to GNNs

Adversarial attack on Graph learning algorithms.

Attacker misleads a learning algorithm (e.g. GNN) into making incorrect predictions or classifications by deliberately perturbing a small number of edges (e.g. remove/add edges) or node features.



Adversarial perturbation (around target node 7) causes misclassification.

Contributions

1. We introduced a novel topological adversarial defense, namely, the *Witness Graph Topological Layer (WGTL)*.
2. WGTL integrates local and global higher-order graph characteristics and controls their potential defense role via a topological regularizer.

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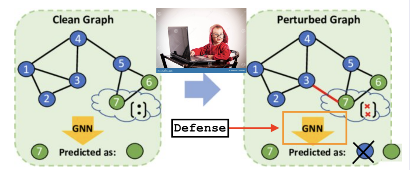

Design a defense algorithm that mitigates the effect of adversarial attack

Contributions

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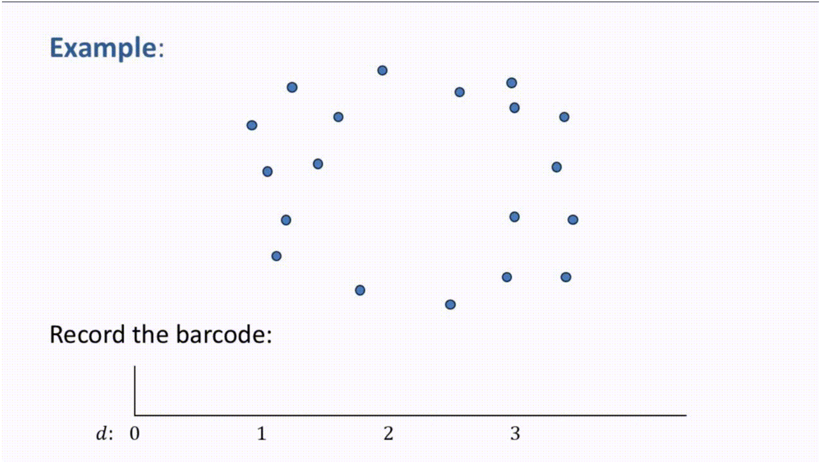
Topological Features



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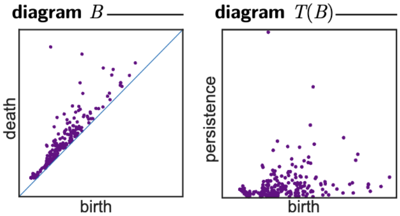

A persistence diagram is transformed using function $T : (x, y) \rightarrow (0, y - x)$.

Why Topological features?

Stability theorem: Small change in the data (graph) only result in small changes in the persistence diagram.

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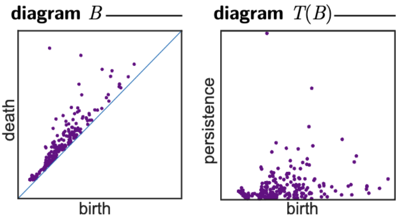

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Impact of this work

1. This is the first work that shows that topological features can make GNNs robust against adversarial perturbations.
2. Effective against a wide variety of attacks, for instance,
   - Targetted poisoning attack (Graybox, modify the neighbors of a target node and their features)
   - Global poisoning attack (Graybox, instead of targetting specific neighborhood modify whichever edges minimizes the model accuracy)
   - Adaptive attacks (White-box, the model architecture, parameters and defense mechanisms are known to the attacker)
   - Node feature attack
3. WGTL improves existing defenses such as Pro-GNN, GNNGuard, and SimP-GCN respectively by 5%, 15%, and 5%.

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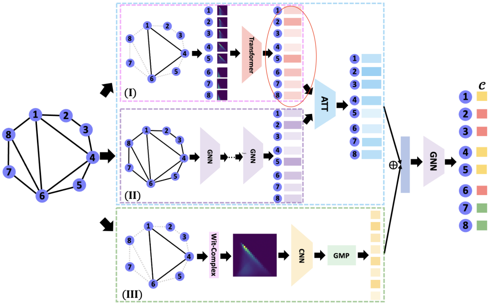

Architecture of Witness Graph Topological Layer.

1. **Local Topology Encoding:** Encodes local topological features of every node. (Z_{T_L})
2. Node Representation Learning. Learns node representations using any backbone GNN. (Z_G)
3. Global Topology Encoding. Encodes topological feature of the entire graph. (Z_{T_G})
4. Aggregated Topological Encoding. Encodes local and global topological priors. (Z_{WGTL})

$$z = [Z_{T_L}, Z_G]$$

$$\text{Attention coefficients, } \alpha_i = \mathbf{Softmax}(\boldsymbol{W_2} \cdot \tanh(\boldsymbol{W_1} z_i + \boldsymbol{b_1}))$$

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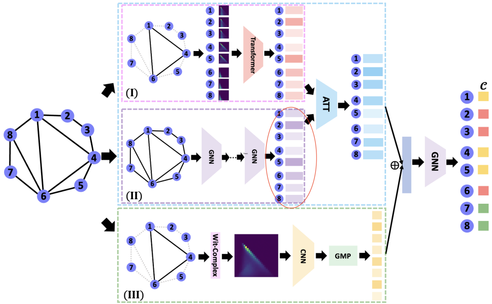
WGTL: Topological Encodings



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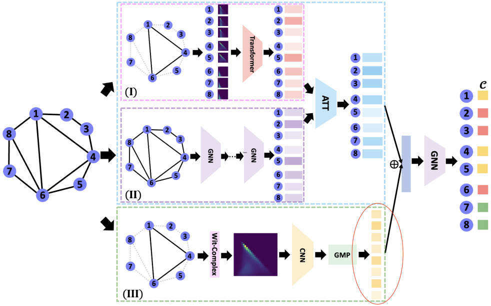

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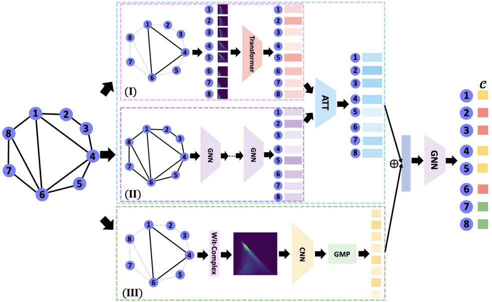

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WGTL: Topological Regularizer

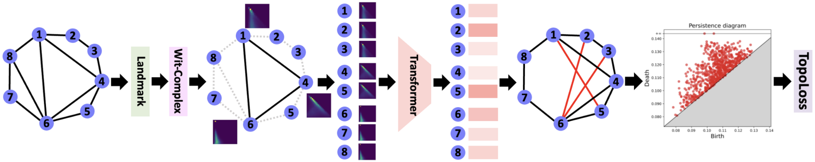
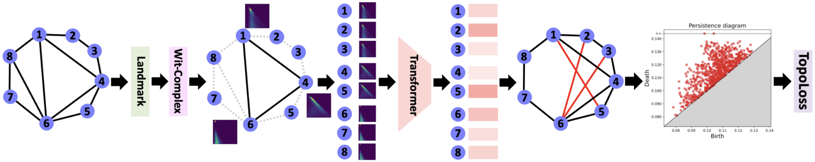


Illustration of Witness Complex-based topological regularizer L_{Topo}.

$$L_{topo}(\mathrm{T}(\mathcal{G})) \triangleq \sum_{i=1}^{m} (d_i - b_i)^2 \left(\frac{d_i + b_i}{2} \right)^2, \tag{1}$$

- A localized attack (perturbing certain nodes or edges) appears as topological noise in the final persistent diagram, and exhibit lower persistence.
- And minimising L_{topo} forces the Transformer to learn local topology encodings (Z_{T_L}) which produces PD with small persistence, i.e., $(d_i - b_i)$.



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Effectiveness

Table 1: Comparison of performances (avg. accuracy±std.) with existing defenses under mettack.

Dataset	Models	Perturbation Rate		
		0%	5%	10%
Cora-ML	Pro-GNN	82.98±0.23	80.14±1.34	71.59±1.33
	Pro-GNN+WGTL	**83.85±0.38**	**81.90±0.73**	**72.51±0.76**
	GCN+GNNGuard	83.21±0.34	76.57±0.50	69.13±0.77
	GCN+GNNGuard+WGTL	*__84.78±0.43__	*__83.23±0.82__	*__79.96±0.49__
	SimP-GCN	79.52±1.81	74.75±1.40	70.87±1.70
	SimP-GCN+WGTL	**81.49±0.52**	**76.65±0.65**	**72.88±0.83**
Citeseer	ProGNN	72.34±0.99	68.96±0.67	67.36±1.12
	ProGNN+WGTL	**72.83±0.94**	**71.85±0.74**	**70.70±0.57**
	GCN+GNNGuard	71.82±0.43	70.79±0.22	66.86±0.54
	GCN+GNNGuard+WGTL	**73.37±0.63**	**72.57±0.17**	**66.93±0.21**
	SimP-GCN	73.73±1.54	73.06±2.09	72.51±1.25
	SimP-GCN+WGTL	*__74.32±0.19__	*__74.05±0.71__	*__73.09±0.50__
Pubmed	Pro-GNN	87.33±0.18	87.25±0.09	87.20±0.12
	Pro-GNN + WGTL (ours)	**87.90±0.30**	*__87.77±0.08__	*__87.67±0.22__
	GCN+GNNGuard	83.63±0.08	79.02±0.14	76.58±0.16
	GCN+GNNGuard+WGTL	OOM	OOM	OOM
	SimP-GCN	*88.11±0.10	86.98±0.19	86.30±0.28
	SimP-GCN+WGTL	OOM	OOM	OOM
Polblogs	GCN+GNNGuard	95.03±0.25	73.25 ±0.16	72.76±0.75
	GCN+GNNGuard+WGTL	*__96.22±0.25__	*__73.62±0.22__	*__73.72±1.00__
	SimP-GCN	89.78±6.47	65.75±5.03	61.53±6.41
	SimP-GCN+WGTL	**94.56±0.24**	**69.78±4.10**	**69.55±4.42**

Efficiency

Table 2: Efficiency of WGTL. All the times are in seconds.

Datasets/ (# Landmarks)	Landmark selection time	Local feat. comput. time	Global feat. comput. time
Cora-ML/124	0.01±0.01	0.12±0.03	5.11±0.13
Citeseer/105	0.01±0.01	0.16±0.02	5.23±1.22
Polblogs/61	0.01±0.00	0.07±0.01	4.64±0.2
Snap-patents/91	0.03±0.02	0.64±0.00	7.54±1.15
Pubmed/394	0.07±0.01	0.51±0.03	27.83±0.47
OGBN-arXiv/84	1.02 ±0.00	12.79±0.31	83.04±2.19