

ϵ -net Induced Lazy Witness Complexes on Graphs

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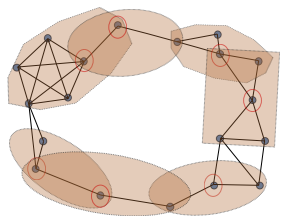
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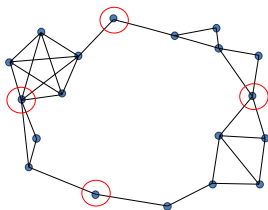
ϵ -net Induced LW Complex on Graph Metric Space

We address the issue of *scalable and fast computation of approximate persistent homologies* using ϵ -nets.

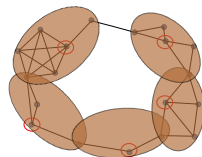
ϵ -net of a graph is an ϵ -sample and ϵ -sparse subset of its vertices.



1-sample : Each vertex is within geodesic 1 from some red vertex



1-sparse: Each pair of red points are at least geodesic 1 away from each other



1-net: Set of points that are 1-sparse and 1-sample

ϵ -net as landmark set induces Lazy Witness filtration that approximates Rips filtration and facilitates faster computation.

Quality of ϵ -net landmarks

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ϵ -net induced LW filtration is a weak 3-approximation of ϵ -net induced Rips filtration.

Theoretical and Algorithmic Contributions

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Algorithmic

We propose three algorithms to construct ϵ -net of a graph.

We experimentally observe that **Iterative ϵ -net** algorithm is efficient in practice.

We want to use ϵ -net...

- to obtain better approximation guarantees for LW filtration;
- to investigate, compare and unify approximation schemes of Rips filtration such as sparse Rips, graph induced complex etc;
- to design scalable and fast algorithms for any metric space;
- to efficiently apply persistent homologies to machine learning and statistical problems.

!! See you at the poster session !!