Hypergraph Drawing by Force-directed Placement

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Hypergraphs

Hypergraph: A finite collection of *set of objects*. The objects are called *vertices*. The sets are called *hyperedges*.

- Hypergraphs can capture multi-ary relationships.
- Hypergraphs generalize graphs (binary relationships).

Applications

- Social Networks: In modeling communities[1], Tagging relationships in music social networks[2].
- Database: In representing Database Schema[3].
- Biology: In representing Yeast protein network[4], Biochemical reaction network[5].

[1] Michael Brinkmeier, Jeremias Werner, and Sven Recknagel. "Communities in graphs and hypergraphs". In: *Conference on information and knowledge management*. ACM. 2007.

[2] Jiajun Bu et al. "Music recommendation by unified hypergraph: combining social media information and music content". In: *Proceedings of the 18th ACM international conference on Multimedia*. ACM. 2010.

[3] Ronald Fagin. "Degrees of acyclicity for hypergraphs and relational database schemes". In: *Journal of the ACM (JACM)* (1983).

[4] Emad Ramadan, Arijit Tarafdar, and Alex Pothen. "A hypergraph model for the yeast protein complex network". In: *Parallel and Distributed Processing Symposium, 2004. Proceedings. 18th International*. IEEE. 2004.

[5] Can Özturan. "On finding hypercycles in chemical reaction networks". In: *Applied Mathematics Letters* (2008).

Hypergraph Visualization Literature

There are two basic methods for drawing a hypergraph.

- **Subset based**:- A hyperedge is drawn as a closed curve **enveloping** its vertices.
- **Edge based**:- A hyperedge is drawn as a set of curves **connecting** its vertices.

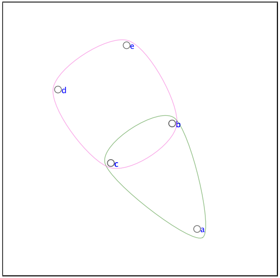


Figure 1: Subset-based drawing.

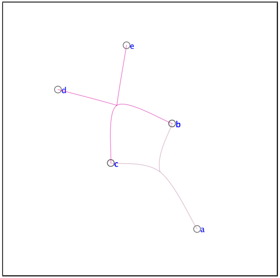
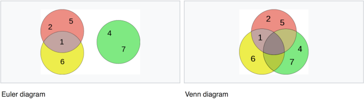


Figure 2: Edge-based drawing.

Set visualization approaches

- Euler diagram, Venn diagram:



  - Venn diagrams are special kind of Euler diagrams (with constraints such as, all possible intersections must be displayed).
  - Euler diagrams are special kind of 'subset based' drawings (with constraints such as, empty zones are not allowed).

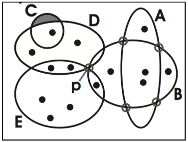


Figure 3: A 'Subset based' drawing. Zones as shaded in C are not allowed in Euler diagram[6].

[6] Rodrigo Santamaría and Roberto Therón. "Visualization of intersecting groups based on hypergraphs". In: *IEICE TRANSACTIONS on Information and Systems* (2010).

Set Visualization approaches (contd.)

- Bubble Sets[7]: Sets are visualized using continuous, **isocontours**.
- LineSets: Sets are visualized using continuous **curves**.

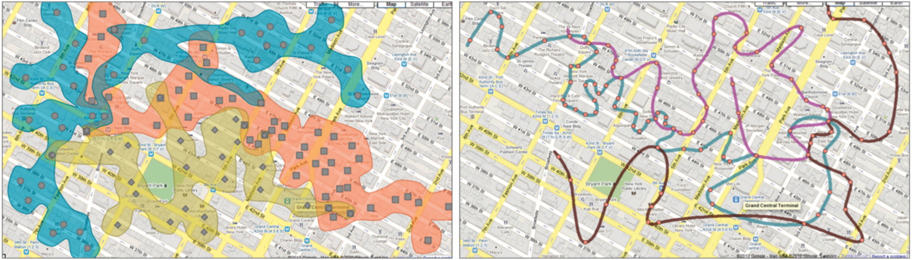


Figure 4: Bubble Sets and LineSets of three set of hotels on the map.[8]

[7] Christopher Collins, Gerald Penn, and Sheelagh Carpendale. "Bubble sets: Revealing set relations with isocontours over existing visualizations". In: *IEEE Transactions on Visualization and Computer Graphics* (2009).

[8] Basak Alper et al. "Design study of linesets, a novel set visualization technique". In: *IEEE transactions on visualization and computer graphics* (2011).

Problem Statement

- We want to have **aesthetically pleasing** drawing of hypergraphs in subset standard.
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  - We propose a family of algorithms.
- We want to evaluate the drawing **quality** by some **measurable criteria**.
  - We propose several metrics.

Algorithms

A Detour to Graph Drawing

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Fruchterman-Reingolds (FR) Force-directed algorithm:

- Vertices: Objects in a physical system.
- Vertices connected (not connected) by edges attract (repel) each other.
- Advantages: Uniform edge length, Symmetry

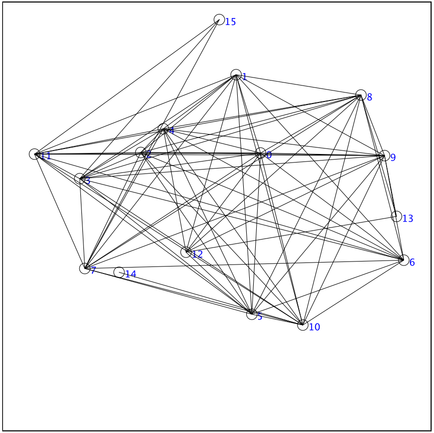

Figure 5: Randomly initialized drawing of a graph.

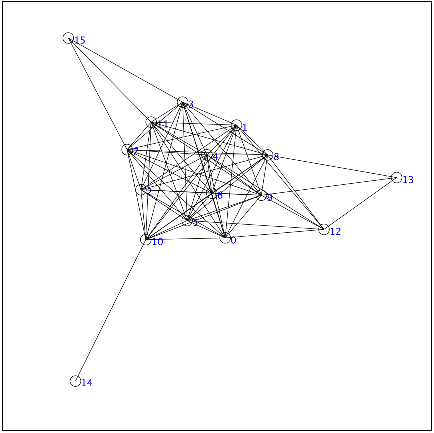

Figure 6: The same graph drawn by FR algorithm.

Algorithm for Hypergraph drawing: Subset based

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Given a Hypergraph, $H = \{\{a, b, c, d\}\}$

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- Transform the Hypergraph to a Graph (namely, the *Associated graph* of a hypergraph).

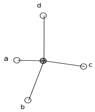

Figure 7: Star Associated graph

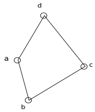

Figure 8: Cycle Associated graph

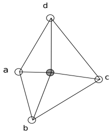

Figure 9: Wheel Associated graph

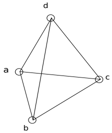

Figure 10: Complete Associated graph

Algorithm for Hypergraph drawing (contd.)

- Each of the transformations induces an algorithm (Star/Cycle/Wheel/Complete algorithm). Draw the Associated graph using FR algorithm (or any Force-directed graph layout algorithm).

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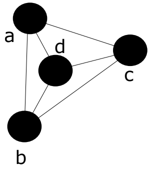

Figure 11: The layout of the *Complete associated graph* after applying FR algorithm.

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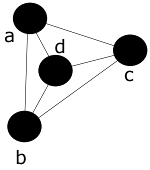
- Draw a closed curve enveloping the vertices of each hyperedge.



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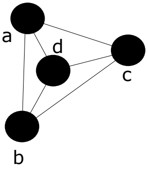

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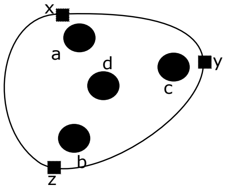

Figure 12: A closed curve is drawn enveloping the vertices.

Criteria of Good Drawings

Planarity

‘Planarity’ refers to the number of non-adjacent hyperedge crossings.
‘Planarity’ should be **minimized**.

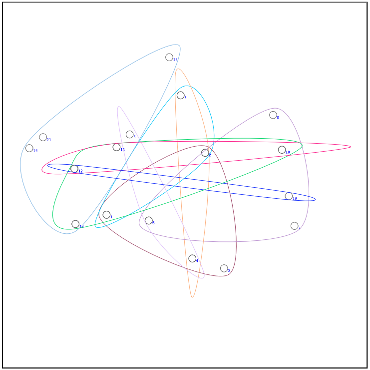

Figure 13: A drawing of a hypergraph with 21 pairwise intersections between non-adjacent hyperedges.

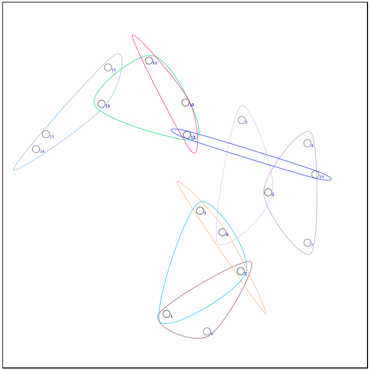

Figure 14: The drawing of the same hypergraph with 2 pairwise intersections between non-adjacent hyperedges.

Coverage

'Coverage' refers to the ratio of the 'mean area per vertex' of the drawing to the 'mean area per vertex' of the entire drawing canvas. 'Coverage' metric should be **maximized**.

$$\text{Mean-APV}_{drawing}(H) = \frac{\sum_{i=1}^{|E|} \frac{Area(E_i)}{|E_i|}}{|E|} \tag{1}$$

$$\text{Mean-APV}_{canvas}(H) = \frac{Area_{canvas}}{|V|} \tag{2}$$

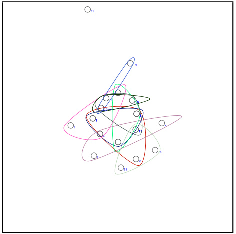

Figure 15: A drawing of a hypergraph with bad 'Coverage' (3.13%).

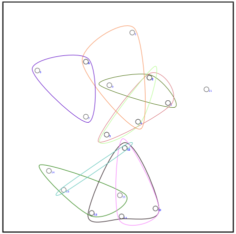

Figure 16: The drawing of the same hypergraph having better 'Coverage' (8.58%).

Regularity

'Regularity' measures how much uniformly spread the vertices are. Drawing canvas is divided into grid and frequency distribution of vertices in each grid is compared with that of a regular placement (Fig. 17). The test statistic (D value) of the Kolmogorov–Smirnov test between these two distribution measures 'Regularity'. 'Regularity' should be **minimized**.

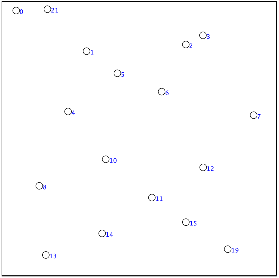

Figure 17: Regular placement of vertices on the drawing canvas.

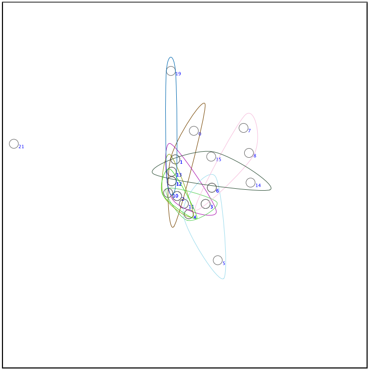

Figure 18: A drawing with bad 'Regularity'.

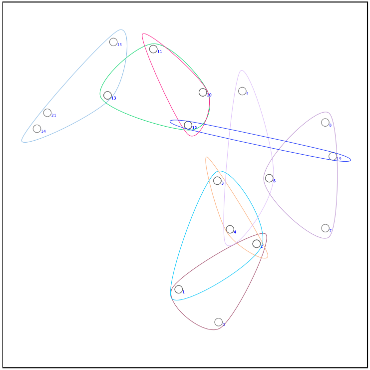

Figure 19: A drawing (of the same hypergraph) with better 'Regularity'.

Datasets and Experiments

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Experimental Results:

- The drawings are rarely Concave since we use only those vertices which are in the convex hull.
- The Complete and the Wheel algorithms have better Planarity than the rest (Reason: The dominance of attractive forces among the vertices in the associated graphs).
- The Cycle and the Star algorithm have better Coverage than the rest (Reason: The dominance of repulsive forces among the vertices in the associated graphs).

Some drawings

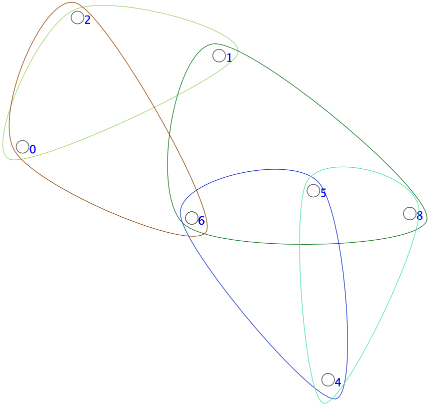

Figure 20: A 3-uniform hypergraph with 5 hyperedge.

Figure 21: A hypergraph with 100 hyperedges randomly sampled from the DBLP dataset

Conclusion

- We propose a family of algorithms for drawing hypergraphs.
- We propose measurable criterion to evaluate the goodness of the drawings.
- We were able to generate aesthetically pleasing drawings.
- Drawbacks: The drawings are not so great in 'Regularity' in cases of hypergraphs with many connected components.
  - Possible improvement(?): Hyperedges modeled as elastic manifolds. (Future work)
- Future work: Perform user study to evaluate the readability of the drawings.

Questions-

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